'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2023

CC14-MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

Time Allotted: 2 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Symbols have their usual meaning.

GROUP-A

Answer any *four* questions $3\times 4 = 12$

1. Determine the region in the xy-plane in which the P.D.E. $(1-x^2)u_{xx} = u_{yy}$ is hyperbolic. 3

2. Find the nature of the P.D.E. and find its characteristic variables: 3

$$u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0.$$

3. Solve $z = px + qy + p^2 + q^2$. 3

4. Define Cauchy problem for one dimensional wave equation. 3

5. Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to canonical form. 3

6. Show that the pedal equation of a central orbit is given by $\dfrac{h^2}{p^3}\dfrac{dp}{dr} = F$. 3

GROUP-B

Answer any *four* questions $6\times 4 = 24$

7. Solve $u_t = c^2 u_{xx}$; $u(0, t) = 0 = u(l, t)$ for all t and $u(x, 0) = f(x)$ for all $x \in [0, l]$. 6

8. By using D'Alembert's principle, solve the following P.D.E. 6

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2};\ -\infty < x < \infty$$

$$u(x, 0) = \sin x;\ \frac{\partial u}{\partial t}(x, 0) = 1.$$

9. Use method of separation of variables to solve 6

$$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,$$

$$u(x, 0) = 4e^{-x}.$$

10. Derive the heat conduction equation. 6

11. Find the integral surface of $pq = xy$ which passes through the curve $z = x$, $y = 0$; by using the method of characteristics. 6

12. A particle is projected with a velocity V from the cusp of an inverted cycloid down the arc. Show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{\sqrt{4ag}}{V}\right)$. 6

GROUP-C

Answer any *two* questions 12×2 = 24

13.(a) Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes through the parabola $4z + x^2 = 0,\ y = 0$. 6

(b) Show that the equations $xp - yq = x$, $x^2p + q = xz$ are compatible and find their solution. 6

14.(a) Eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z = yf(x) + xg(y)$, obtain the P.D.E. $xys = px + qy - z$. 6

(b) Deduce D'Alembert's formula of the Cauchy problem 6

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2};\ -\infty < x < \infty,\ t > 0 \text{ subject to the conditions:}$$

$$u(x,0) = f(x),\ u_t(x,0) = g(x) \text{ for } -\infty < t < \infty.$$

15.(a) Solve by Lagrange's method 4

$$py + qx = xyz^2(x^2 - y^2).$$

(b) State Cauchy-Kowalevski theorem and prove it for the following problem: 2+6

$$\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = F(x,t),\ 0 < x < l,\ t > 0$$

subject to the conditions

$$u(x,0) = f(x),\ u_t(x,0) = g(x) \text{ for } 0 \le x < l \text{ and } u(0,t) = u(l,t) = 0,\ t \ge 0.$$

16.(a) Find the temperature distribution in a rod of length l. The faces are insulted and the initial temperature distribution is given by $x(l - x)$. 5

(b) Give an example of a quasi-linear P.D.E. Discuss the method of characteristic to solve the following quasilinear P.D.E. 1+4+2

$$Pp + Qq = R$$

and also show that the Lagrange's auxiliary equation is given by

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

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